The research of modeling of natural convection in metal solidification process with finite different method was conducted to determine temperature distribution and fluid flow profile with variations value Rayleigh number. The research conducted by solving governing equation of natural convection with finite difference approximation. Governing equation of natural convection consist of continuity equation, momentum equations, and energy equation. The ADI (Alternating Directional Implicit) method was used to discrizeze for governing equation of natural convection. Finite difference method was written in Fortran language whereas the temperature distribution and fluid flow profile were visualized with Matlab software. The results of this research was validated by comparing the results obtained with Rajiv Sampath research. Comparison of the results of research showed good agreement. The result showed that solidification process occurs faster at Ra $10^4$ compared with $10^5$ and $10^6$.

1. INTRODUCTION
Numerous things related to the heat transfer are found in everyday life, especially in the field of industry. Heat transfer can occur in three ways, namely conduction, convection, and radiation. Convection is a heat transfer that occurs between solid surface to the moved fluid which caused by the temperature difference within. Convection which is based on the fluid flow origins are categorized into two categories, namely the forced convection and natural convection.

Forced Convection is heat transfer fluid flow convection which happened influenced by external tools, such as fans, pumps, and others. While natural convection is heat transfer fluid flow convection which is caused by the differences in fluid density caused by heating and cooling. Natural convection plays an important role in the engineering industry, one of them in the metal solidification process. Research which concerned on natural convection freezing problem is extremely crucial, because the fluid flow which caused by natural convection in liquid state, it changing the shape of the liquid/ solid interface and temperature distribution during freezing (Yinheng, 1994).

Physical phenomena that control the solid/ liquid interface shape during freezing are becoming necessary in numerous industrial processes. Its main characteristic is that the interface moves to separate the two phases with different physical properties. Differences in temperature cause the buoyancy liquid produces significant convection currents. Natural convection has a major influence on the morphology of its interface, freezing rate, and temperature distribution (Mohammad 2009).

Research about the natural convection issue of metals freezing has been carried out both experimental and numerically. Experimental laboratory research requires a significant financial cost and the process is quite complicated. Therefore, numerically study was developed which much cheaper. Various methods of numerical approach to determine the natural convection phenomenon has been done, using a mathematical model of the continuity equation, momentum equation, and energy equation.


2. SCOPE
This study is aimed to make a natural convection modeling in the metal solidification process with finite difference methods. The velocity vector and temperature distribution were included.
3. LITERATURE REVIEW
McDaniel and Zabaras (1994) made a 2D numerical modeling on the basis of natural convection phase transformation on the freezing and thawing issues of pure metal using the finite element method. There were two cases analyzed. It was 105 and 106 of Rayleigh numbers. The boundary conditions that used were top, bottom, and right side insulation while the left side was the convection.

Chen and Yoo (1995) analyzed the freezing process aluminum in natural convection with the finite element method.

Sampath and Zabaras (1999) created 2D and 3D numerical modeling on the basis of natural convection phase transformation using the finite element method. Sampath and Zabaras Research investigated the freezing and thawing problem of pure metals and alloys. Basically, this study was continued by Zabaras research (1994).


4. RESEARCH METHODOLOGY
4.1 Research Procedure
Research was done by making the program implementation to resolve the momentum equation, energy equation, and the continuity equation with the ADI method.

An outline of the research can be made following flow chart:

4.2 Diskritisasi Persamaan Atur
Natural convection equation set consists of the continuity equation, momentum equation, and energy equation. In order to set the natural convection equations can be applied in the programming language. Firstly, discretization equation set was set up.

This study implementation, the discretization equation solved by the method set ADI. By defining

\[
\frac{1}{2}(x_{i,j}^n + x_{i+1,j}^n) = XR, \quad \frac{1}{2}(x_{i,j}^n + x_{i-1,j}^n) = XL, \\
\frac{1}{2}(x_{i,j}^n + x_{i,j+1}^n) = XT, \quad \frac{1}{2}(x_{i,j}^n + x_{i,j-1}^n) = XB \]

where \( x \) is unknown variable (such as, \( u, v, \) and \( \theta \)).

4.2.1 Momentum Equation Discrete
4.2.1.1 X-axis Momentum Equation
Discretization of momentum equation used x-axis momentum equation x direction without including the pressure element. Therefore, the equation becomes:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{Pr}{Ra^{0.5}} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + Pr \cos \theta
\]

For example \( \frac{Pr}{Ra^{0.5}} \), so, the equation above become,
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + A \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \text{Pr} \frac{\partial \theta}{\partial \varphi} \]

**a. X-Sweep**

X-Sweep equation for momentum equation x-axis is:

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + A \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \text{Pr} \frac{\partial \theta}{\partial \varphi} \]

Discretization for each equation above can be explained:

- \[ \frac{\partial u}{\partial t} = \frac{u_{ij}^{n+1/2} - u_{ij}^n}{\Delta t} \]
- \[ u \frac{\partial u}{\partial x} = \frac{1}{2 \Delta x} \left( \text{UR} u_{ij}^{n+1/2} + (\text{UR-UL}) u_{ij}^{n+1/2} + \text{UL} u_{ij}^{n+1/2} \right) \]
- \[ \frac{\partial^2 u}{\partial x^2} = \frac{u_{ij}^{n+1/2} - 2 u_{ij}^n + u_{ij}^{n-1/2}}{\Delta x^2} \]
- \[ \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial y^2} \]

By substituting equations above and multiplying with \( \frac{\Delta t}{2} \), can be obtained matrix tridiagonal coefficient:

- \( \frac{\partial u}{\partial t} = \frac{\partial u}{\partial t} \]
- \( u \frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} \]
- \( \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial x^2} \]

**b. Y-Sweep**

Y-Sweep equation for momentum equation y-axis is:

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + A \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \text{Pr} \frac{\partial \theta}{\partial \varphi} \]

Discretization for each equation above can be explained:

- \[ \frac{\partial u}{\partial t} = \frac{u_{ij}^{n+1/2} - u_{ij}^n}{\Delta t} \]
- \[ u \frac{\partial u}{\partial x} = \frac{1}{2 \Delta x} \left( \text{UR} v_{ij}^{n+1/2} + (\text{UR-UL}) v_{ij}^{n+1/2} + \text{UL} v_{ij}^{n+1/2} \right) \]
- \[ \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial x^2} \]
- \[ \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial y^2} \]

By substituting equations above and multiplying with \( \frac{\Delta t}{2} \), can be obtained matrix tridiagonal coefficient:

- \( \frac{\partial u}{\partial t} = \frac{\partial u}{\partial t} \]
- \( u \frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} \]
- \( \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial x^2} \]

**4.2.1.2 Y-axis Momentum Equation**

Discretization of momentum equation used y-axis momentum equation y direction without including the pressure element. Therefore, the equation becomes:

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \text{Pr} \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \text{Pr} \frac{\partial \theta}{\partial \varphi} \]

For example \( A = \frac{\text{Pr}}{\rho_0} \), so, the equation above becomes:

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \text{A} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \text{Pr} \frac{\partial \theta}{\partial \varphi} \]

**a. X-Sweep**

X-sweep equation for momentum equation x-axis is:

\[ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + A \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = -v \frac{\partial v}{\partial y} + A \frac{\partial^2 v}{\partial x^2} + \text{Pr} \frac{\partial \theta}{\partial \varphi} \]

Discretization for each equation above can be explained:

- \[ \frac{\partial u}{\partial t} = \frac{u_{ij}^{n+1/2} - u_{ij}^n}{\Delta t} \]
- \[ v \frac{\partial u}{\partial x} = \frac{1}{2 \Delta x} \left( \text{UR} v_{ij}^{n+1/2} + (\text{UR-UL}) v_{ij}^{n+1/2} + \text{UL} v_{ij}^{n+1/2} \right) \]
- \[ \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial x^2} \]
- \[ \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial y^2} \]
By substituting equations above and multiplying with \( \frac{\Delta t}{2} \), can be obtained: tridiagonal matrix coefficient:

\[
\begin{align*}
a_i &= \frac{\Delta t}{2\Delta x} \left[ \frac{UL}{\Delta x} + \frac{A}{\Delta x} \right] \\
b_i &= \left[ 1 + \frac{\Delta t}{4\Delta x} (UR-UL) + \frac{A\Delta t}{2\Delta x^2} \right] \\
c_i &= \frac{\Delta t}{2\Delta y} \left[ \frac{VB}{\Delta y} + \frac{A}{\Delta y} \right] \\
d_i &= u_{i+1/2} - \frac{\Delta t}{2\Delta x} (UR-VR-UL.VL) \\
&+ \frac{A\Delta t}{2\Delta x} \left( v_{i+1/2} - 2v_{i+1/2} + v_{i-1/2} \right) + \frac{\Delta t}{2} \Pr \theta \cos \phi 
\end{align*}
\]

4.2.2 Pressure Iteration Using Line Gauss-Seidel

Formula was used to calculate pressure is:

\[
\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y} = \frac{1}{\Delta t} \left( \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \right)
\]

Discretization for each equations above:

\[
\frac{\partial^2 p}{\partial x^2} = \frac{p_{i+1,j} - 2p_{i,j} + p_{i-1,j}}{\Delta x^2}
\]

\[
\frac{\partial^2 p}{\partial y^2} = \frac{p_{i,j+1} - 2p_{i,j} + p_{i,j-1}}{\Delta y^2}
\]

4.2.3 Energy Discretization Equation

Energy discretization equation was used:

\[
\frac{\partial \theta^2}{\partial x^2} + \frac{\partial \theta^2}{\partial y^2} = \frac{1}{\Delta t} \left( \frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial y} \right)
\]

For example \( \frac{1}{Ra^{0.5}} \), so, the equation above becomes:

\[
\frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial y} = B \left( \frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial y} \right)
\]

From those equations above, could produce discretization:

4.2.3 Y-Sweep

Y-sweep equation for momentum equation y-axis is:

\[
\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + A \frac{\partial^2 v}{\partial x^2} = -u \frac{\partial v}{\partial y} + \frac{A}{\Delta y} \Pr \theta \cos \phi
\]

Discretization for each equation above can be explained:

\[
\frac{\partial v}{\partial x} = \frac{v_{i+1,j} - v_{i-1,j}}{2\Delta x}
\]

\[
\frac{\partial^2 v}{\partial y^2} = \frac{v_{i+1,j+1} - 2v_{i+1,j} + v_{i+1,j-1}}{\Delta y^2}
\]

\[
\frac{\partial^2 v}{\partial x^2} = \frac{v_{i+1,j+1} - 2v_{i+1,j} + v_{i+1,j-1}}{\Delta y^2}
\]

By substituting equations above and multiplying with \( \frac{\Delta t}{2} \), can be obtained matrix tridiagonal coefficient:

\[
\begin{align*}
a_i &= \frac{\Delta t}{2\Delta y} \left[ \frac{VB}{\Delta x} + \frac{A}{\Delta x} \right] \\
b_i &= \left[ 1 + \frac{\Delta t}{4\Delta x} (VT-VB) + \frac{A\Delta t}{2\Delta x^2} \right] \\
c_i &= \frac{\Delta t}{2\Delta y} \left[ \frac{VB}{\Delta y} + \frac{A}{\Delta y} \right] \\
d_i &= \frac{\Delta t}{2\Delta y} \left( v_{i+1/2} - 2v_{i+1/2} + v_{i-1/2} \right) + \frac{\Delta t}{2} \Pr \theta \cos \phi
\end{align*}
\]
\[ v \frac{\theta}{\partial y} = \frac{1}{2\Delta y} \left[ VT \cdot \theta_{i+1,j} + (VT \cdot VB) \cdot \theta_{i,j}^{n+1} + \theta_{i-1,j} \right] \]
\[ \frac{\partial \theta}{\partial y} = \frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{\Delta y^2} \]
\[ u \frac{\theta}{\partial x} = \frac{1}{\Delta x} \left[ UR \cdot 0 - UL \cdot 0 \right] \]
\[ \frac{\partial^2 \theta}{\partial x^2} = \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{\Delta x^2} \]

By substituting equations above and multiplying with \( \frac{\Delta t}{2} \), can be obtained matrix tridiagonal coefficient:
\[ a_i = \frac{\Delta t}{2\Delta y} \left[ UL \cdot B \right] \]
\[ b_i = \frac{\Delta t}{4\Delta y} \left( VT \cdot VB \right) + \frac{B \Delta t}{2\Delta y^2} \]
\[ c_i = \frac{\Delta t}{2\Delta y} \left[ VB \cdot B \right] \]
\[ d_i = \theta_{i+1,j} - \frac{\Delta t}{2\Delta x} \left( UR \cdot 0 - UL \cdot 0 \right) + \frac{B \Delta t}{2\Delta x^2} \left( \theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j} \right) \]

5. RESULT AND DISCUSSION
5.1 Program Validation

Validation program was done by comparing the current research to the Rajiv Sampath research (1999). Rajiv Sampath research domain (1999) was the resolution of natural convection case in metal solidification process on a square mold with 1:1 of aspect ratio, with the walls below, above, and right side condition were insulating, while the left Wall was convection.

![Boundary and require research condition](image)

Figure 5.1 Boundary and require research condition

Rajiv Sampath research (1999) was using finite element method with the same boundary conditions to the present study boundary conditions. Isothermal visualization and velocity vector results will be compared with Rajiv Sampath research (1999) at Ra = 105. Isothermal comparison results is shown in following Table 4.1. and Table 4.2:

<table>
<thead>
<tr>
<th>Titik</th>
<th>Sekarang Grid</th>
<th>Grid Beda (%)</th>
<th>Sampath Beda (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>Y</td>
<td>61x61</td>
<td>81x81</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3</td>
<td>0.52931</td>
<td>0.50247</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3</td>
<td>0.65937</td>
<td>0.65371</td>
</tr>
<tr>
<td>0.7</td>
<td>0.2</td>
<td>0.82506</td>
<td>0.81854</td>
</tr>
<tr>
<td>0.8</td>
<td>0.3</td>
<td>0.92960</td>
<td>0.92603</td>
</tr>
</tbody>
</table>

Table 4.1. Isothermal comparison result at t=20

Table 4.2. Isothermal comparison result at t=55

Isothermal comparison results between the current study to Sampath research above shows good accuracy with 5862 for the 61x61 grid and 2892 for 81x81 of a maximum error. It shows for the 81x81 grid has a maximum error value is smaller than the 61x61 grid, so that in this study we used the 81x81 grid. Visualization comparison of nowdays research with Sampath research above can be shown in the following figure:
Natural Convection Simulation on Metal Solidification Process

Natural convection simulation cases on metal solidification process on a square mold is shown with the 81x81 grid, 0.0149 of Prandlt number (Pr), and dt= 0.001 of time step, and Rayleigh number (Ra) variations were 104,105 and 106. The simulation results can be seen in the following image:
Temperature transformation can be seen in the temperature distribution graph which served by a point \((x, 0.1)\), wherein \(x\) is 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1. Temperature distribution graph is as follows:

This also occurred on \(Ra= 105\) and \(106\) which shown in Figure 4.6 and Figure 4.8. Isothermal figure which can be seen that the fluid movement on the wall to get the cooling fluid density will be increased. The fluid density escalation caused the fluid to move downwards, while the fluid has a lower density will move upwards. Fluid movement downward was also influenced by gravity however fluid movement upward influenced by their buoyancy occurs due to it has a smaller density.

Isothermal plot results in Figure 4.4, Figure 4.6, and Figure 4.8 portrays the temperature distribution visually. Figure 4.4 at \(Ra = 104\) displays that the cold fluid moves down and the hot fluid moving upwards. Cold fluid movement influenced their gravity and density changes due to its temperature changes which is producing the density increased, while the hot fluid movement was affected by buoyant force because it has a smaller density than the cold fluid. Fluid temperature near the left wall was strongly influenced by the ambient temperature.
Velocity vector visual results with the velocity vector variation can be seen in Figure Ra 4:10 to 4:12. It portrays that the fluid movement opposite to clockwise and freezing the bottom left looks faster. It also illustrates that the cold fluid moves down and the hot fluid moving upwards. Cold fluid movement was influenced by their gravity and density changes due to its temperature transformation which caused the density went up, while the movement of hot fluid was affected by Buoyancy force due to it has a smaller density than the cold fluid.

c. Rayleigh number greatly affects the metal solidification speed. The smaller of Rayleigh, the faster the solidification process and the greater Rayleigh number clots more slowly.

6. Conclusion
From this research and discussion that has been done. It produced a numerous conclusions:

a. Research comparison of Rajiv Sampath study (1999) for natural convection problem in metal solidification process on a square mold indicates that the method which was used in this study can provide acceptable results in those cases.

b. Cold fluid moved down and the hot fluid moving upwards. Cold fluid movement was influenced by their gravity and density changes due to its temperature transformation which caused the density went up, while the movement of hot fluid was affected by Buoyancy force due to it has a smaller density than the cold fluid.

c. Rayleigh number greatly affects the metal solidification speed. The smaller of Rayleigh, the faster the solidification process and the greater Rayleigh number clots more slowly.

7. Reference